Study on Hemispherical Soft-Fingered Handling for Fine Manipulation by Minimum D.O.F. Robotic Hand

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Abstract—This paper provides that an elastic force and an elastic potential energy due to the deformation of soft fingers, which are previously derived, can be calculated into straightforward equations in an analytical way. These formulae lead to the fact that the potential energy of a soft fingertip is a function of two variables, and has a local minimum through the elastic rolling of a contacting object. We formulate four geometric constraints in grasping motions of a rigid object by means of two degrees of freedom robotic hand that has two rotational joints. We define a basic motion including translational and rolling motions when two fingers rotate by infinitesimal angle, and propose a quasi-static manipulation and its algorithm by using the local minimum of elastic potential energy (LMEE) of soft fingers with geometric constraints. In this theory, we define an energy function included in the LMEE algorithm. By solving that function we simulate the path of the center of gravity and the change of orientation of the grasped object, and compare those values with measurements experimentally obtained from a CCD camera equipped above the manipulated object. Finally, we confirm the effectiveness of the quasi-static manipulation theory based on the LMEE algorithm by performing handling experiments of a grasped object.

II. LOCAL MINIMUM OF ELASTIC POTENTIAL ENERGY (LMEE)

We have previously proposed the static elastic model and its elastic potential energy equation on a hemispherical soft fingertip which occur in the case of large deformation of the fingertip. We have numerically described a local minimum of elastic potential energy (LMEE) which arises in that deformation. These equations, however, remain double integration forms which are difficult to intuitively understand the existence of the LMEE. In this section, we show these physical quantities in straightforward forms as follows:

\[ ||\mathbf{F}|| = \frac{\pi Ed^2}{\cos \theta_p}, \]
\[ P = \frac{\pi Ed^3}{3\cos^2 \theta_p}, \]

Fig. 1. Elastic force and potential energy models

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where $E$ means the Young’s modulus of a soft fingertip material, and $d$ and $\theta_p$ denote the maximum displacement of the soft fingertip and the orientation angle of a contacting object respectively, as shown in Fig.1. The derivation process of eq.(2) is detailed in Appendix.

Eqs.(1) and (2) indicate that the total elastic force is proportional to the square of maximum displacement and in inverse proportion to the cosine of orientation angle of the object. Also, the elastic potential energy is proportional to the cube of maximum displacement and in inverse proportion to the square of cosine of the object angle. Note that both equations have a local minimum when the orientation angle takes zero.

III. GEOMETRIC CONSTRAINTS

In this paper, we propose a new manipulation theory based on LMEE algorithm by considering the handling motion of a grasped object in a quasi-static condition. First, we categorize geometric constraints into two directions: vertical and tangential directions, and formulate these constraints in the following.

Fig. 2. Soft-fingered manipulation

A. Normal Constraint

As illustrated in Fig.2, we consider a soft-fingered handling by means of minimum D.O.F. robotic hand that has two rotational joints. We assume that the robotic hand grasps a planar rigid object in two-dimensional plane, and ignore the gravitational force in this system.

Let $a$ be the radius, $d_i$ and $d_j$ be the right and left maximum displacements of soft fingertips, $L$ be the length of fingers, and $W_{obj}$ be the object width of a grasped object, respectively. Let $2W_{fi}$ be the width between foots of both fingers, and $2d_{fi}$ be the thickness of that finger. Furthermore, let $\theta_i$ and $\theta_j$ be the rotational angle of both fingers, $O_r$ and $O_l$ be the fingertip origin, and $R$ be the origin of reference coordinate system $\Sigma_R$. Letting $\theta_{obj}$ be the orientation angle of the object with respect to $\Sigma_R$, Each coordinate of points $O_r$ and $O_l$ is described with respect to $\Sigma_R$ as follows:

$$O_r = \begin{bmatrix} W_{fi} - L \sin \theta_i - d_{fi} \cos \theta_i \\ L \cos \theta_i - d_{fi} \sin \theta_i \end{bmatrix}$$

(3)

$$O_l = \begin{bmatrix} -W_{fi} + L \sin \theta_j + d_{fi} \cos \theta_j \\ L \cos \theta_j - d_{fi} \sin \theta_j \end{bmatrix}$$

(4)

where $\theta_{obj}$ takes positive value in counter-clock-wise direction. As shown in Fig.3-(a), let $\Sigma_{obj}$ be the object coordinate system having the origin $C$, and $G(x_{obj}, y_{obj})$ be the center of gravity of the grasped object with respect to $\Sigma_{obj}$, whose coordinate equals to $(w, h)$ in $\Sigma_{obj}$. Letting $A$ be the center of contacting circular area, and $O_{A_j}$ be the foot of a perpendicular dropped on a line $OA_j$, the length of $O_jQ_j$ can be represented as follows:

$$O_jQ_j = (x_{obj} - O_{ix}) \cos \theta_{obj} + (y_{obj} - O_{iy}) \sin \theta_{obj}.$$  

(5)

Thus, we derive a geometric normal constraint in the left fingertip when the plane contact occurs with the maximum displacement $d_j$.

$$(x_{obj} - O_{ix}) \cos \theta_{obj} + (y_{obj} - O_{iy}) \sin \theta_{obj} = (a - d_j) + \frac{W_{obj}}{2} + w.$$  

(6)

As well as the left fingertip relation, we obtain another equation as illustrated in Fig.3-(b).

$$-(x_{obj} - O_{rx}) \cos \theta_{obj} - (y_{obj} - O_{ry}) \sin \theta_{obj} = (a - d_j) + \frac{W_{obj}}{2} - w.$$  

(7)

where $(O_{rx}, O_{ry})$ and $(O_{lx}, O_{ly})$ denote $xy$-coordinates in eqs.(3) and (4).

B. Rolling Constraint

In order to formulate rolling constraints in terms of the tangential direction on both fingertips, we assume that each fingertip does not slip along the surface of the grasped object. As illustrated in Fig.4-(a), let
\( \theta_{obj}^{-1}, \theta_{obj}^{0}, \ldots, \theta_{obj}^{-1}, \theta_{obj}^{0} \) be the object orientation when the object iterates \( n \)-times rolling motions on the fingertip, and let us use \( \theta_{obj}^{-1} \) instead of \( \theta_{obj}^{0} \) as a matter of convenience in the following. Let us draw a straight line passing points \( O_I \) and \( A \), let \( P \) be the foot of a perpendicular dropped on the center line passing point \( C \) from the center of gravity \( G \). \( Q_I \) be the intersection on that perpendicular, and each point is defined as \( P(w, 0) \) and \( Q_I(\omega, h) \) with respect to the coordinate system \( \Sigma_{obj} \). Here, \( s_I \) is a relative position with respect to the tangential direction between point \( C \) and the fingertip, as shown in Fig.4-(a). That is, \( s_I \) corresponds to a physical quantity including the past history of object rolling distance. Consequently, when the object rolls on the fingertip with radius \( a - d_I \), the length \( GQ_I \) is updated by adding the rolling distance \( AA' \) as follows:

\[
GQ_I = h - s_I - (a - d_I) \cdot (\theta_{obj} - \theta_{obj}^{-1}),
\]  

(8)

where the counter-clock-wise direction associated with the object orientation \( \theta_{obj} \) is positive as illustrated in Fig.4. At the same time, a geometric relationship on the tangential direction shown in Fig.3-(a) is described as below:

\[
GQ_I = -(x_{obj} - O_x) \sin \theta_{obj} + (y_{obj} - O_y) \cos \theta_{obj}.
\]  

(9)

Finally, from eqs.(8) and (9) the rolling constraint on left fingertips is expressed as follows:

\[
-(x_{obj} - O_x) \sin \theta_{obj} + (y_{obj} - O_y) \cos \theta_{obj} = h - s_I - (a - d_I) \cdot (\theta_{obj} - \theta_{obj}^{-1}),
\]  

(10)

As well as the above equation, the rolling constraint of the object on the right fingertip is written by

\[
-(x_{obj} - O_x) \sin \theta_{obj} + (y_{obj} - O_y) \cos \theta_{obj} = h - s_I + (a - d_I) \cdot (\theta_{obj} - \theta_{obj}^{-1}).
\]  

(11)

Here \( s_I \) denotes \( y \)-coordinate of point \( Q_I(\omega, h) \) with respect to the coordinate system \( \Sigma_{obj} \), and is a physical quantity including the past rolling distance on the right fingertip as well as \( s_I \) shown in Fig.4-(b). Eqs.(10) and (11) are geometric tangential constraints due to the object rolling on both fingertips.

IV. QUASI-STATIC HANDLING MOTION

In this paper, we do not consider the dynamics of fingers, fingertips and the grasped object, and confine to a quasi-static handling motion. We define that an instantaneous single process for manipulating motions be a minimum basic motion in which both infinitesimal rolling and translational motions of the object are combined during the handling process. In the following section, by using the concept of LMEE due to the large deformation of soft fingertips, we propose a quasi-static handling algorithm in which physical variables \((x_{obj}, y_{obj}, \theta_{obj})\) are determined uniquely after minimum basic motions.

A. Quasi-Static Handling Algorithm with LMEE

We develop the concept of LMEE represented in eq.(2) to a minimum D.O.F. robotic hand with two rotational joints. As shown in Fig.2, the total elastic potential energy due to the deformation of both fingertips are represented as follows:

\[
P = \frac{\pi E}{3} \left\{ \frac{d_I^3}{\cos^2(\theta_{obj} - \theta_{obj})} + \frac{d_I^3}{\cos^2(\theta_{obj} + \theta_{obj})} \right\}.
\]  

(12)

We find \((d_I, d_I')\) and \((y_{obj}, y_{obj}')\) such that eq.(12) reaches its local minimum, where all variables are uniquely determined by using the rolling constraints eqs.(10) and (11) and the normal constraints eqs.(6) and (7). The procedure of the quasi-static handling algorithm proposed in this paper is summarized as follows:

1) Finger joint angles \((\theta_{obj}, \theta')\) are arbitrarily given as input angles.
2) An LMEE of eq.(12) is computed.
3) The center of gravity of a grasped object \((x_{obj}, y_{obj})\) and its orientation angle \(\theta_{obj}\) are determined from the LMEE value.
4) Each maximum displacement of both fingertips $(d_r, d_l)$ is calculated from the above computed values $(x_{obj}, y_{obj}, \theta_{obj})$.

Here the grasped object is rigid and its dimension is an arbitrary rectangular solid. We assume that the slip motion between the object and the soft fingertip is ignorable and all manipulation and grasping processes in this system are confined to the two-dimensional plane without the gravity effect.

V. SIMULATION

We search for a local minimum of eq.(12) by using a numerical algorithm, Nelder-Mead Method, in every infinitesimal change of the finger rotation.

![Fig. 5. Quasi-static manipulation process](image)

(a) Initial state (b) Operation 1 (c) Operation 2 (d) Operation 3

We summarize the simulation procedure in terms of quasi-static handling motion as follows:

1) As shown in Fig.5-(a) and (b), both fingers rotate by a certain degree to the inner side from an initial condition $\theta_r = \theta_l = d_r = d_l = 0$ (Operation 1). In this paper we perform manipulation tests for five different grasping angles in the operation 1, where it takes 2.4 deg up to 7.2 deg at the interval of 1.2 deg.

2) Both fingers rotate toward the counter-clock-wise (CCW) direction by 20 deg as shown in Fig.5-(c) (Operation 2).

3) Both fingers rotate toward the clock-wise (CW) direction by 40 deg as shown in Fig.5-(d) (Operation 3).

4) Both fingers rotate toward the CCW direction by 40 deg.

5) Operation 3 and 4 iterate three times.

6) Both fingers rotate toward the CW direction by 20 deg and return back to the operation 1.

7) Both fingers go back to the initial state.

All parameters used in this simulation equal to those of a robotic hand fabricated for handling experiments. Also, we set that the center of gravity of the object $G$ and the geometric center $C$ are identical to each other for simplicity. Every simulation result is plotted in experimental results shown in the following chapter.

VI. EXPERIMENTS

In this paper we conduct quasi-static handling experiments by means of a minimum D.O.F. robotic hand, and compute the center of gravity and object orientation by a CCD camera in real time. We validate the quasi-static handling theory based on LMEE by comparing simulations and experiments.

![Fig. 6. Apparatus](image)

A. Apparatus

As shown in Fig.6, we conduct the quasi-static handling experiment by using stepping motors and hemispherical soft fingertips mounted on two rotational fingers. The diameter of the fingertip is 40 mm and the grasped object has 49 mm square. We perform this experiment on the two-dimensional plane, and all parameters relating to the robotic hand are shown in Table I. The Young’s modulus of soft fingers is measured by the tensile test of a polyurethane rubber.

B. Experimental Method

We attach a rectangular black drawing paper on the object, and compute the center of gravity and the orientation of the object by using a CCD camera equipped above the robotic hand. We compute the luminance center of the extracted binary images of the black drawing paper as a center of gravity of the object, and then obtain an object orientation angle by computing the second order moment of the binary images shown in Fig.7.
C. Experimental results

Fig. 8 shows experimental results in which simulation results are also plotted together. Left column shows the path of center of gravity \((x_{\text{obj}}, y_{\text{obj}})\) during grasping and manipulation, and right column shows the change of object orientation with respect to \(x_{\text{obj}}\). The first row graphs correspond to the results of grasping angle 2.4 deg in the operation 1. Also, five different experimental results in terms of grasping angle up to 7.2 deg are shown in Fig. 8. Here the origins in every graph are equivalent to those of the center of black drawing paper attached on the grasped object shown in Fig. 6.

The position of the object shown in the left column goes downward largely, which results from the fact that the object follows to the large deformation of soft fingertips because of the frictional restriction of the contact area between the object and fingertips in the experiment. In the manipulating motion, we can find that both simulation and experimental results are identical to each other as well as the grasping motion.

On the other hand, the orientation results shown in the right column in Fig. 8 indicate that a S-shaped curve appears explicitly in every graph in experiments. This results from the fact that each rolling distance on both fingertips increases as the rolling motion progresses and the rolling radii \((d_i - d_{i+1} \rightarrow d_{i+1})\) get larger.

Fig. 9 shows a comparison between all experimental results associated with the path of the object position. The most important point is that each end point of the path of manipulation processes conforms closely to all together at approximately ±28 mm. In this experiment, every rotational angle of both fingers \((\theta_i, \theta_{i+1})\) after operation 2 and 3 is all different in five patterns of grasping angle.
from 2.4 deg to 7.2 deg. This resultant consistency of $x_{obj}$ implies that the $x$-coordinate of the object position does not depend on the magnitude of the grasping force. In other words, in order to transfer an object to a certain position along the $x$-axis, it is only necessary to activate the fingers by a certain degree that corresponds one-to-one with the position of the object irrespective of the elastic force level of soft fingertips.

VII. CONCLUDING REMARKS

In this paper we have proposed a quasi-static handling theory based on LMEE algorithm due to the large deformation of a hemispherical soft fingertip. Also, we have two normal constraints and two rolling constraints including variable rolling radius on a soft fingertip, and shown that each of $(x_{obj}, y_{obj}, \theta_{obj})$ of a grasped object is uniquely determined by numerical analysis using the LMEE algorithm.

APPENDIX

As illustrated in Fig.1, since the elastic potential energy caused by the deformation of soft fingertip is represented as a double integration on $xy$-plane, it takes the following equation by using the spring constant $k$ of a virtual linear spring $QR$:

$$P = \frac{1}{2} \int_{S} kPQ^2 = \frac{1}{2} \int_{S} k(QT^2 + PT^2).$$

Using the geometric relationship of the triangle $PQT$ illustrated in Fig.10, the above equation is transformed into the following equation:

$$P = \frac{1}{2 \cos^2 \theta_p} \int_{C} kQT^2,$$

where $C$ denotes the circular region. The above form can finally be calculated in an analytical way as eq.(2).

REFERENCES